

where  $\theta = \varepsilon_x + \varepsilon_y + \varepsilon_z$ . In uniaxial strain,  $\varepsilon_x = (V_0 - V)/V_0$ ,  $\varepsilon_y = \varepsilon_z = 0$ . If the equality holds, the material is in the plastic state.

ii) In the plastic state, every increment in strain is the sum of an elastic and a plastic increment:

$$(43a) \quad d\varepsilon_x = d\varepsilon_x^e + d\varepsilon_x^p,$$

$$(43b) \quad d\varepsilon_y = d\varepsilon_y^e + d\varepsilon_y^p,$$

$$(43c) \quad d\varepsilon_z = d\varepsilon_z^e + d\varepsilon_z^p.$$

iii) There is no plastic dilatation:

$$(44) \quad d\varepsilon_x^p + d\varepsilon_y^p + d\varepsilon_z^p = 0.$$

iv) The stress is supported solely by the elastic strain:

$$(45a) \quad dp_x = \lambda d\theta + 2\mu d\varepsilon_x^e,$$

$$(45b) \quad dp_y = \lambda d\theta + 2\mu d\varepsilon_y^e,$$

$$(45c) \quad dp_z = \lambda d\theta + 2\mu d\varepsilon_z^e,$$

where  $\lambda$  and  $\mu$  are, in general, functions of the density.

As  $p_x$  is increased from zero, the response is initially elastic and  $\varepsilon_y = \varepsilon_z = 0$ . Then

$$(46) \quad p_x - p_y = (1 - 2\nu)p_x/(1 - \nu),$$

where  $\nu = \lambda/2(\lambda + \mu)$  is Poisson's ratio. The yield stress is reached at a value of  $p_x$  called the «Hugoniot elastic limit», denoted by  $p_{\text{HEL}}$ . From eqs. (41) and (46):

$$(47) \quad p_{\text{HEL}} = (1 - \nu)Y/(1 - 2\nu).$$

For further increases in  $p_x$ , the material is in the plastic state. Then

$$(48) \quad p_x \equiv \bar{p} + \frac{2}{3}(p_x - p_y) = \bar{p} + 2Y/3,$$

where  $\bar{p} = (p_x + p_y + p_z)/3$ , a function of density and internal energy alone. Referring to Fig. 14 b), eq. (48) applies to the segment  $AB$  of the  $p_x$  curve. The slope of the  $(p_x, V)$  curve in the elastic region is, from eqs. (42):

$$(49) \quad dp_x/dV = -(\lambda + 2\mu)/V_0 = -(K + 4\mu/3)/V_0,$$

where  $K$  is bulk modulus. In the plastic region,  $AB$ , the slope is, for constant  $Y$ , from eq. (48)

$$(50) \quad dp_x/dV = d\bar{p}/dV = -K/V.$$

In accord with eq. (50), it is convenient to define the incremental dilatation as  $dV/V$ . Bulk modulus normally increases with  $\bar{p}$ , so  $AB$  is normally concave upward. The yield stress,  $Y$ , is in general a function of plastic work and density. In such case eq. (50) is augmented by a  $dY/dV$  term. In any case the offset of  $p_x$  from the hydrostat,  $\bar{p}$ , is always  $2Y/3$ .

At point  $B$  in Fig. 14 *b*) we suppose that a change is made from monotonically increasing to monotonically decreasing  $p_x$ . Equation (41) must again be examined to determine whether the mass element is in the elastic or plastic state. During the initial compression process,  $p_x$  increased more rapidly than  $p_v$  until yield occurred. During unloading,  $p_x$  decreases more rapidly than  $p_v$  until yielding again occurs. Thus the portion  $BC$  of the unloading curve is elastic until  $p_v - p_x = Y$  at  $C$ . From  $C$  to  $D$ , unloading is plastic and the unloading curve lies below the hydrostat by  $\frac{2}{3}Y$ .

Referring to the discussion following eq. (17), we see that point  $A$  of Fig. 14 *b*) may be a point of instability for single shock compressions. To see that this is indeed the case, suppose that a shock wave has been generated with amplitude  $p_{\text{HEL}}$ , traveling with speed

$$D_E = [V_0(\lambda + 2\mu)]^{\frac{1}{2}}.$$

The velocity of this shock front relative to the material behind it is

$$(51) \quad D_E - u_E = (V_A/V_0)D_E = V_A \sqrt{(\lambda + 2\mu)/V_0}.$$

If an additional compression of small amplitude is produced to follow the already established shock, it will travel with velocity  $c_A$  relative to the material ahead of it, where, according to eq. (50),

$$c_A = \sqrt{KV_A} = V_A \sqrt{(\lambda + 2\mu/3)/V_A}.$$

Comparing this with eq. (51) we find that

$$(52) \quad (D_E - u_E)^2/c_A^2 = (3V_A/V_0)(1 - \nu)/(1 + \nu) \simeq 3(1 - \nu)/(1 + \nu) = \frac{3}{2} \quad \text{for } \nu = \frac{1}{3},$$

since  $V_A/V_0 \simeq 1$  at the Hugoniot elastic limit. According to eq. (52), the second wave does not overtake the shock, so there is a region of the  $(p_x, V)$  curve above the point  $A$  which cannot be reached by a single shock from